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Unsymmetrical Buckling of Shallow Spherical Shells

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THE numerical results for the critical pressure for unsymmetrical buckling of clamped, shallow, spherical shells recently were presented by Weinitschke.[1] The author of the present note independently has obtained results for the same problem which are in striking disagreement with those of Weinitschke. The governing differential equations used agree with those used by Weinitschke. The buckling pressures were calculated numerically. The final results are shown in Fig. 1 together with some available experimental data.[2, 3] The pressure parameter p is defined as the ratio of the external pressure q to the classical buckling pressure q_0 of the complete spherical shell of the same radius of curvature and thickness; n is the number of waves along the circumferential direction appearing in the buckling mode. The disagreement with Weinitschke's results is displayed in Fig. 2, but the reason for this disagreement remains unknown.

According to the author's results, unsymmetrical buckling ($n \neq 0$) occurs only for $\lambda > 5.5$. For $\lambda < 5.5$, buckling occurs by axisymmetrical snapping. As λ keeps increasing, the buckling mode shows more and more waves along the circumferential direction and also shows a distinct boundary layer near the edge of the shell along the radial direction when λ is high. An asymptotic value of the buckling pressure is found to be 0.864 when λ approaches infinity and the ratio of n/λ approaches 0.817. Initial imperfections of the shell are presumed to be the source of the discrepancy between theory and experiment.

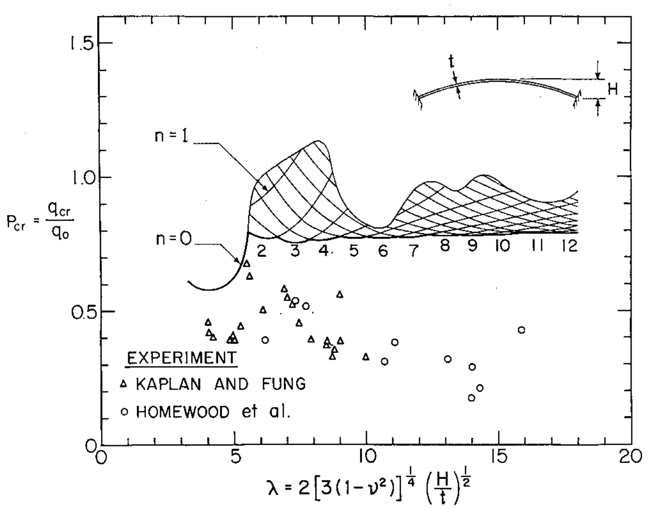

Fig. 1 Calculated buckling pressures of clamped, shallow, spherical shells and experimental results

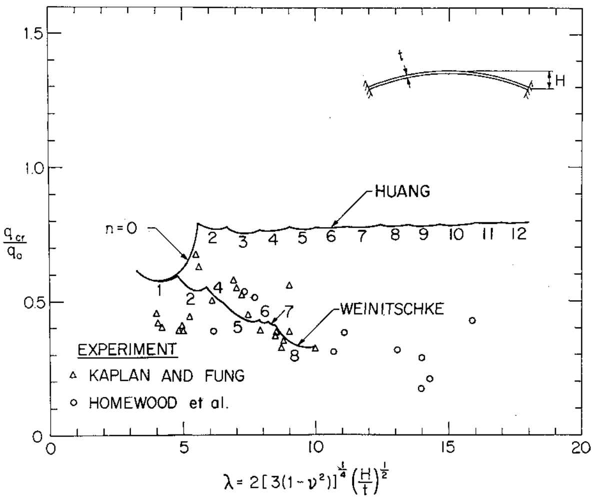

Fig. 2 Calculated buckling pressures vs Weinitschke's results

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On Axially Symmetric, Turbulent, Compressible Mixing in the Presence of Initial Boundary Layer

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RECENT experimental results[1] have shown that the mixing of heterogeneous gases having an initial velocity ratio close to unity occurs faster than is predicted by classical eddy-viscosity theory. The theoretical analysis of two uniform streams of different gases but of nearly equal velocity, performed with the usual assumptions for eddy viscosity and Prandtl number equal to a constant,[2] shows that mixing will take place very slowly, i.e., at the rate corresponding to laminar diffusion. It has been suggested that the difference

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